Vibration and Flutter of Stringer-Stiffened Cylindrical Shells

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Results are presented which show that a stiffened shell can have lower natural frequencies than the corresponding unstiffened shell. Flutter studies of such shells show that lower critical Mach numbers may also result under certain circumstances. Results are presented for linear piston theory and exact potential theory which show convergence and confirm that the former theory is acceptable provided the critical Mach number is sufficiently high and especially if the shell is short and the critical circumferential wave number is high.

Nomenclature

a_o = speed of sound
E = Young's modulus
h = shell thickness
h_s = stringer depth
k = frequency parameter = $\omega L/U$
k_1 = $2(1-\nu^2)(R/L)(R/h)(\rho_o U^2/E)$
L = shell length
M = Mach number
m = number of axial half waves
n = number of circumferential waves
Q_{mn} = nondimensional generalized aerodynamic force
q_{mn} = generalized coordinate
R = shell radius
U = air velocity
u, v, w = shell middle surface displacements in x, y, z directions
x, y, z = axial, circumferential, radial coordinates
ρ_c; ρ_o = material density; air density
ω = frequency
ν = Poisson's ratio
Δ = $\rho_c R^2\omega^2(1-\nu^2)/E$

Subscripts
F = flutter
mn = mnth in-vacuo mode
a = assumed
p = predicted
crit = critical (for flutter)
min = minimum (of natural frequency)

Introduction

A COMPREHENSIVE review[1] of the literature on shell flutter covering some 140 publications has concluded that there is a need for an intensive study of the effects of various structural forms and idealizations and aerodynamic assumptions on shell flutter.

The majority of past studies have considered isotropic circular cylindrical shells of finite length supported on rigid end rings or bulkheads and in no case have stringer-stiffened circular cylindrical shells apparently been studied, despite their wide applicability in aerospace structures. However ring and stringer stiffened conical shells have been studied[2] and for the configurations studied, the addition of either rings or stringers increased the flutter speed.

Although intuitively one might expect unstiffened shells to be more flutter-prone, such shells are often, in practice, internally pressurized for structural stabilization. Also, recent studies[3,4] have shown that the addition of internal or symmetric stringer stiffening to an unpressurized, unstiffened shell can cause a decrease in the natural frequencies of those shell modes pertinent to shell flutter. The possibility was also posed[4] of a consequent decrease in the predicted flutter speed.

The present paper summarizes the results obtained in Ref. 5 which examined the above possibility and gave flutter convergence studies based on both linear piston theory and exact potential flow theory. Comparative studies of the generalized forces for shell flutter resulting from various aerodynamic theories have recently been reported by the authors[6,7] and comments made on the range of geometrical parameters, etc., for which linear piston theory can be expected to be applicable.

Throughout this paper, it is assumed that the shells have simply-supported ends and are reinforced by a set of discrete, equally spaced uniform stringers integrally attached to the shell surface.

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Vibration Studies

The theoretical analyses of Ref. 4 included the effects of inplane and rotary inertia of the shell and stringer configuration and the effects of stringer eccentricity from the shell median plane. The additional effect of "smearing" the stringers over the entire shell circumference was also considered.

The modes assumed for the shell displacements were taken as

$$\begin{aligned} u &= \bar{u}\cos(m\pi x/L)\cos(ny/R)\sin\omega t \\ v &= \bar{v}\sin(m\pi x/L)\sin(ny/R)\sin\omega t \\ w &= \bar{w}\sin(m\pi x/L)\cos(ny/R)\sin\omega t \end{aligned} \tag{1}$$

where $\bar{u}$, $\bar{v}$, $\bar{w}$ are the modal amplitudes in the axial, circumferential, and radial directions, ω is the circular frequency, m is the number of axial half waves, and n is the number of circumferential full waves.

Using Eqs. (1) in the Rayleigh-Ritz method, a cubic frequency determinant results in the nondimensional frequency parameter, Δ. Of the three frequencies determined for a particular set of shell and stringer parameters, the minimum corresponds to essentially radial vibrations.

Table 1 gives theoretical results for a shell stiffened by four internal stringers which was previously considered in Refs. 3 and 8. The present results are slightly lower probably due to the retention of inplane inertias. The assumption of "discrete" stiffening as in Ref. 3 appears to have little advantage over "smeared" stiffening even for sparse stiffening configurations. The agreement with experiment[8] is good and it is clearly seen that the addition of stringers has decreased the natural frequencies.

Corresponding theoretical and experimental studies have been made[5] for a shell fabricated from commercially available aluminum sheet with a longitudinal bonded lap joint. Stringer stiffeners of rectangular strip section have been bonded to the external surface of the shell at equal circumferential spacing.

Table 1 Frequencies of a shell stiffened with four internal stringers[3] ($m = 1$)

n	Present study Discrete	Smeared	Experiment[8]	Egle and Sewall[3] Symmetric	Antisymmetric	Unstiffened
2	314.61	315.31	...	...	...	...
3	158.72	158.72	...	169	169	171
4	100.27	102.21	100	103	108	108
5	93.09	93.09	87	95	95	98
6	115.00	113.91	104	109	116	117
7	144.00	144.00	137	145	145	151
8	179.71	185.26	176	183	192	194
9	233.26	233.26	224	236	236	243
10	296.81	287.38	295	278	297	300

Eccentricity of the stiffeners was increased by bonding additional rectangular strips to the existing stiffeners.

An electromechanical shaker was used to excite the shell and a microphone to measure the radial vibrations. An automatic scanning device enabled axial and circumferential traverses to be made readily.

The corresponding experimental and theoretical results for the minimum natural frequencies for a particular shell are given in Table 2. This shell was made of 0.052 in. thick aluminum, 28 in. long, and 28 in. diam. Strips 0.25 in. wide from the same material were used to form the 12 equal stringers.

Good agreement has been shown in general between the experimental results and theoretical predictions for the unstiffened shell for $m = 1$ for excitation frequencies between 150 and 1000 Hz corresponding to values of n between 7 and 20. In some other cases, the responses obtained were poor or inconclusive and the identification of modes was difficult. This was particularly so where $m = 1$ and $m = 2$ frequencies were close. Very good responses were achieved when $n = 12$ (number of stringers).

It is clearly seen that as the number of strips, and hence, the mass and the external eccentricity increases, the minimum natural frequency first decreases because of the added mass and then increases as a result of the increasing eccentricity and stiffening. However for higher frequencies, even with increasing external eccentricity, the greater influence of increasing mass tends to cause a continuing decrease in these frequencies. This must be because of the inefficient form of the stringers used which differ considerably from those used in practical shell-stringer configurations. It should be noted that the stringer height-to-width ratio used in Refs. 3 and 8 and Table 1 was about 10, whereas in the present study and Table 2, the maximum value of this ratio was only 2 (with 10 strips). The corresponding added masses of the stringers used on the shells of Tables 1 and 2 were, respectively, 7% and 30% of the unstiffened shells.

Aerodynamic Studies

In Refs. 6 and 7 comparative studies have been reported of the generalized aerodynamic forces for shell flutter based on various theoretical approaches including linear piston theory (LPT), quasisteady theory (QST), slender body theory (SBT), exact potential theory (EPT), and modified exact potential theory (MEPT). Table 3 gives typical results.

It was concluded that LPT can be expected to be a reasonable approximation to EPT or MEPT for short shells vibrating with a large number of circumferential modes, n, at very high Mach numbers. For this range of applicability there is no significant difference between LPT and QST.

The agreement between SBT and EPT appears to exist only for large values of L/R and n and is improved at low M. There is no realistic comparison between SBT and LPT or QST.

Because of the amount of computer time needed to determine the aerodynamic generalized forces from EPT in a flutter analysis and because of the simplicity of LPT, it was decided to assess the relative merits of both theories in a range of flutter studies with the hope that LPT might prove acceptable for a wide range of parameters.

Flutter Analysis

Based on the assumed modes of Eqs. (1) and applying Lagrange's equation, the following flutter determinant is obtained

$$\begin{vmatrix} \Delta_{1n}^* & k_1Q_{12} & \cdots & k_1Q_{1m'} \\ k_1Q_{21} & \Delta_{2n}^* & \cdots & k_1Q_{2m'} \\ \cdots & \cdots & \cdots & \cdots \\ k_1Q_{m'1} & k_1Q_{m'2} & \cdots & \Delta_{m'n}^* \end{vmatrix} \begin{vmatrix} q_{1n} \\ q_{2n} \\ \cdots \\ q_{m'n} \end{vmatrix} = 0 \tag{2}$$

where

$$\Delta_{mn}^* = \Delta_{mn} + k_1Q_{mm} - \Delta_F$$

Table 2 Frequencies of a shell stiffened with twelve external stringers[5] ($m = 1$)

n	Unstiffened T[a]	E	+2 strips T	E	+3 strips T	E	+4 strips T	E	+7 strips T	E	+10 strips T	E
5	207	...	207	...	208	...	210	...	231	225?	233	...
7	166	148?	164	154?	165	178?	167	172	189	172?	192	157?
9	218	220	213	214	212	212	213	209	225	...	227	195
11	311	315	302	...	300	...	298	...	300	...	301	...
12	367	370	346	354	336	340	328	340	302	...	301	...
13	427	437	417	427	412	420	409	418	400	409	400	401
15	569	578	551	564	545	557?	539	552	520	538?	518	526
17	729	743	706	724	697	715	689	708	657	...	655	...
19	909	927	880	...	869	...	858	891?	813	...	809	...

[a] T—Theory; E—Experiment; ?—Doubtful results; ... values not found.

Table 3 Generalized aerodynamic forces Q_{mi}, $n = 9$, $k = 1$[a]

			$L/R = 2$							$L/R = 4$					
M	Q_{11}^R	Q_{11}^1	Q_{12}^R	Q_{12}^1	Q_{22}^R	Q_{22}^1	M	Q_{11}^R	Q_{11}^1	Q_{12}^R	Q_{12}^1	Q_{22}^R	Q_{22}^1	
12	−0.3019	0	0	0.1481	−1.1244	0	2	−0.1510	0	0	0.0741	−0.5622	0	SBT
	0	0.0415	0.1115	0	0	0.0415		0	0.1925	0.7698	0	0	0.1925	QST
	0	0.0417	0.1112	0	0	0.0417		0	0.2500	0.6667	0	0	0.2500	LPT
	−0.0154	0.0447	0.1194	−0.0049	0.0045	0.0375		−0.1566	0.0029	0.0021	0.0812	−0.6073	0.0107	EPT
14	−0.3019	0	0	0.1481	−1.1244	0	3	−0.1510	0	0	0.0741	−0.5622	0	SBT
	0	0.0356	0.0955	0	0	0.0356		0	0.1547	0.4714	0	0	0.1547	QST
	0	0.0357	0.0952	0	0	0.0357		0	0.1667	0.4444	0	0	0.1667	LPT
	−0.0089	0.0374	0.0998	−0.0038	0.0065	0.0319		−0.1603	0.0056	−0.0008	0.0981	−0.6993	0.0555	EPT
16	−0.3019	0	0	0.1481	−1.1244	0	4	−0.1510	0	0	0.0741	−0.5622	0	SBT
	0	0.0312	0.0835	0	0	0.0312		0	0.1205	0.3443	0	0	0.1205	QST
	0	0.0313	0.0833	0	0	0.0313		0	0.1250	0.3333	0	0	0.1250	LPT
	−0.0051	0.0321	0.0855	−0.0031	0.0074	0.0276		−0.1740	0.0138	0.0425	0.1297	−0.7096	0.2160	EPT

[a] EPT = Exact Potential Theory; SBT = Slender Body Theory; QST = Quasi-static Theory; and LPT = Linear Piston Theory.

where

$$k_1 = 2(1-\nu^2)(R/L)(R/h)(\rho_o U^2/E)$$

is a nondimensional aerodynamic stiffness ratio parameter, and Δ_F is the nondimensional flutter frequency parameter where

$$\Delta_F = \rho_c R^2 \omega_F^2 (1-\nu^2)/E$$

Δ_{mn} are the corresponding nondimensional modal in-vacuo frequency parameters in terms of the ω_{mn}.

Δ_F is the flutter eigenvalue and the flutter condition is reached only if Δ_F is real and positive and the predicted Mach number is equal to the assumed value, M_a. In general, the aerodynamic forces can only be calculated by assuming values for M_a and k_a, the frequency parameter. The corresponding set of eigenvalues Δ from Eq. (2) yields results for ω_p and since $k_a = \omega_p L/U$ a predicted Mach number M_p is obtained from $M_p = U_p/a_o = \omega_p L/k_a a_o$. For arbitrarily assumed values of M_a, k_a it is generally found that $M_p \neq M_a$. Therefore, various assumed pairs of values of M_a, k_a are taken until a positive, real eigenvalue Δ_F is obtained and hence a value of M_p. This procedure is repeated until $M_p = M_a$ and the corresponding value of k_a is the critical value of the flutter frequency parameter k_F.

The shell flutter problem has been programed for digital computer solution and numerical results have been obtained for various values of cylindrical shell and flow parameters. For all analyses a steel shell was assumed and a sea-level altitude.

Results and Discussion

Binary Analysis using LPT for Unstiffened Shell

Figure 1 shows the variation of critical Mach numbers as a function of n for a particular value of h/R. The minimum M_{crit} occurs for $n > 0$, in fact, at a value of n_{crit} greater than that corresponding to the minimum natural frequency for $m = 1$ (i.e., n_{min}), as can be seen from Fig. 1. The value of n corresponding to the minimum flutter frequency is also different from n_{crit}. In fact, for $h/R = 0.002$, $L/R = 2.0$, the respective values are 10 and 16 with $n_{min} = 8$.

For very short shells, the value of n_{crit} may be greater than 16. Because of this, the application of EPT for comparison purposes is then more difficult.

The influence of shell thickness on the flutter boundaries is shown in Fig. 2 to be very significant for short shells, less so for long shells.

In Figs. 1 and 2 the dotted lines correspond to the neglect of aerodynamic damping and, clearly, significant errors are introduced in the region of n_{crit} by this omission, particularly for long shells.

Table 4 summarizes some of the previously mentioned results for selected shell parameters.

It should be pointed out that Ref. 9 indicates that binary analyses can give considerable overestimates of the effects of aerodynamic damping.

Binary Analysis using LPT for a Stringer-Stiffened Shell

Table 4 presents results for shells stiffened internally or externally by 10 stringers of rectangular section whose total mass is assumed to be equal to the total mass of the unstiffened shell.

For all the L/R and h/R ratios considered the shells stiffened with external stringers appear to yield much higher M_{crit} values than the comparable internal stringer case.

Eccentricity effects are defined by the ratio (h_s/h) of stringer depth to shell thickness. Provided this ratio is greater than 5.0 for external stringers or 8.0 for internal stringers, large eccentricity appears to yield higher values of M_{crit} for shells of

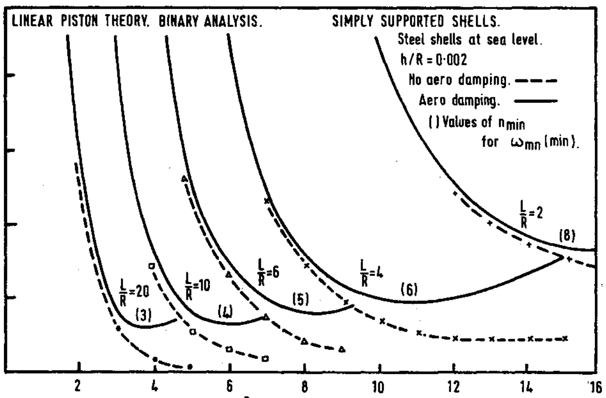

Fig. 1 Flutter Mach number vs n.

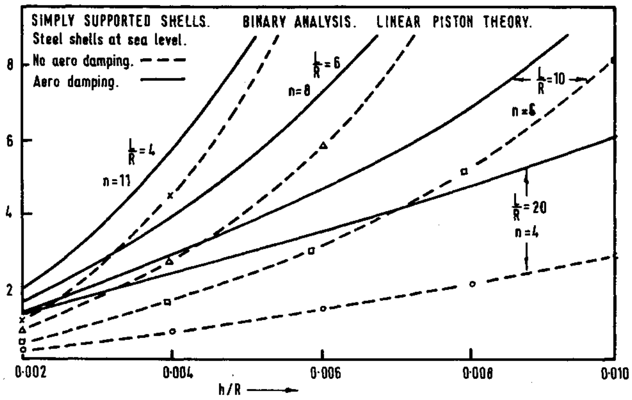

Fig. 2 Flutter Mach number vs h/r.

Table 4 Critical Mach numbers for unstiffened and stiffened shells—binary analysis

		Unstiffened		LPT Internal stringers				External stringers			
h_s/h		0		1		10		1		10	
L/R	h/R	n_{crit}	M_{crit}	n_{crit}	M_{crit}	n_{crit}	M_{crit}	n_{crit}	M_{crit}	n_{crit}	M_{crit}
2	0.002	16	3.35	16	2.37	8	21.1	18	3.43	10	36.47
	0.004	14	14.11	13	9.87	7	97.7	15	16.19	7	175.5
	0.008	11	69.35	10	48.09	5	457.8	11	85.28	6	806.1
	0.012	9	178.9	8	124.6	5	1234.0	10	226.1	4	2021.0
4	0.002	11	1.98	11	1.46	7	6.94	11	1.72	8	10.53
	0.004	10	5.37	10	3.89	6	28.82	11	5.24	7	49.30
	0.008	9	22.54	8	16.38	5	105.4	9	24.60	6	207.1
	0.012	8	57.34	7	40.67	4	234.4	8	64.40	6	481.0
6	0.002	8	1.64	8	1.26	7	3.81	9	1.39	7	5.23
	0.004	8	3.70	8	2.76	5	15.22	9	3.34	6	23.42
	0.008	8	12.34	7	9.14	5	53.23	8	12.34	5	94.20
	0.012	7	29.72	7	22.03	4	99.40	7	31.36	5	205.0
10	0.002	6	1.40	6	1.08	6	1.98	6	1.14	6	2.39
	0.004	6	2.82	6	2.12	5	7.00	6	2.34	5	9.47
	0.008	6	6.88	6	5.14	4	21.76	6	6.20	5	36.53
	0.012	5	14.36	6	10.85	4	40.60	6	13.52	4	74.53

equal mass, at least for short shells. If the eccentricity is small, however, it seems that a monocoque shell of double the thickness yields values of M_{crit} higher than for the corresponding stringer-stiffened shell of the same total mass. As L/R increases, the effectiveness of eccentric stiffening is only observed when the ratio h_s/h is very high.

The results for $h_s/h = 1.0$ correspond to a stringer-stiffened shell of twice the mass (and twice the equivalent thickness) of the basic unstiffened shell. The values of M_{crit} are always less than those of the corresponding unstiffened shell of twice the mass (and thickness), because the present analysis has neglected the circumferential bending and membrane stiffnesses of the individual stringers.

Multimode Analysis using LPT and EPT

Results have also been obtained for a long unstiffened shell with the parameters $L/R = 10$, $n = 6$, $h/R = 0.002$, and 0.004.

Table 5 shows the comparison of the two-mode solution using LPT and EPT with various higher-order solutions up to 10. It can be seen that, for the cases analyzed with LPT, convergence is reached by taking six or seven axial modes.

It is also seen that the increase in number of axial modes when using LPT in Case C does not alter the nondimensional flutter frequency parameters Δ_F significantly, most of them being close to the in-vacuo frequency factor Δ_{m6} corresponding to $m = 2$. An examination of the real eigenvector components q_{m6} corresponding to the flutter solution with 10 axial modes suggests that the predominant influence on flutter is from the modes $m = 2$, 1, and 3 in that order.

The convergence of the results using EPT in Case A appears to be more rapid than for LPT in Case B. Interestingly, for only two modes, the LPT result for M_{crit} is lower than for EPT, but the converged results show the opposite.

Results for a stiffened shell are also shown under Case B and these are directly comparable to those for the unstiffened shell of Case B on the basis of equal total mass. The convergence of the results is good and it is seen that the stiffened shell is marginally better than the equivalent unstiffened shell (the reverse was true on the basis of the binary analysis).

It was stated earlier that LPT should be more valid for smaller values of L/R and larger values of n and M. Table 5 gives comparable results for LPT and EPT, and it may be deduced that the disagreement is slight for $h/R = 0.004$ binary or converged results, but for $h/R = 0.002$, the binary results are quite disparate. This is primarily a consequence of the fact that M_{crit} is so small.

It can probably be concluded, therefore, that LPT is an acceptable alternative to EPT, provided that M_{crit} is sufficiently high.

Conclusions

Theoretical and experimental studies of stringer-stiffened shells have shown that the addition of stringers to an unstiffened shell can cause a decrease in the natural frequencies of those

Table 5 Flutter data for a long shell $L/R = 10$, $n = 6$[a]

				Axial mode no(s)									
Case		h/R		1	2	3	4	5	6	7	8	9	10
A	EPT	0.004	M_{crit}	...	3.10	3.12	3.13	3.20	3.20	3.20	3.20	3.20	3.20
		0.002	M_{crit}	...	2.70	...	...	...	...	...	...	...	...
B	LPT	0.004	M_{crit}	...	2.82	3.48	3.60	3.75	3.75	3.75	3.75	3.75	3.75
		+0.002	M_{crit}	...	1.98	2.75	3.00	3.60	3.65	3.85	3.80	3.90	3.80
C	LPT	0.002	M_{crit}	...	1.40	1.73	1.67	1.82	1.80	1.80	1.80	1.80	1.80
			$\Delta_{m6} \times 10^3$	0.44	0.55	0.98	2.08	4.29	7.86	13.32	20.87	30.69	42.83
			$\Delta_F \times 10^3$	...	0.49	0.57	0.58	0.59	0.59	0.59	0.59	0.59	0.59
			q_{m6}	425.7	617.1	317.9	90.5	29.2	11.0	5.3	2.7	1.6	1

[a] All above results are for unstiffened shells except + which has 10 internal stringers of equal mass to basic shell; i.e., B results are for shell structures of equal mass.

shell modes relevant to shell flutter particularly if the stringers are fixed internally and are not of efficient form.

Aerodynamic studies have suggested that linear piston theory may be an acceptable alternative to exact potential theory provided that the M_{crit} and n are both high and the shell is short.

Flutter studies have confirmed: 1) aerodynamic damping may be very important; 2) external stiffening can cause higher M_{crit} values than internal stiffening; 3) if stiffener eccentricity is small a monocoque shell of double the thickness yields values of M_{crit} higher than for the corresponding stringer-stiffened shell of the same total mass; 4) convergence studies using linear piston theory and exact potential theory yield converged results with only a few axial modes which are not too dissimilar provided that the predicted M_{crit} is sufficiently high; and 5) for one case studied, the stiffened shell is only marginally more stable than the equivalent unstiffened shell. It is also surmized that, in some cases, a stiffened shell may be less stable, but this would probably be so only if the stringer form is inefficient.

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